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Convergence of Three Binomial Models into Black Scholes Model in Establishing Option Prices in Hongkong, India, and Indonesia

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Abstract

The establishment of option prices is one of the crucial aspects in derivative trade. Black-Scholes model (BSM) is one of the most popular models of option price establishment. The option exchanges such as CBOE, HKEX, and NSE use this model to determine the price options. Black-Scholes Model is modelled with stock price movement as a stochastic process. Another popular model is binomial model (BSM), originated from stock exchange movement model which divides interval time [0, T] into n equal length step. It holds several models to determine the value of up-move, down-move, and probability. Binomial model is categorised as Cox-Ross-Rubinstein, Jarrow-Rudd, and Leisen-Reimer. There are numerous literatures which discuss the relation between BM and BSM, including the convergence of binomial model and BSM. The former's model which is often put side-by-side with BSM is Cox-Ross-Rubinstein. Even though this model is quite simple, it requires a lot, even thousands of steps to render Cox-Ross-Rubinstein to converge with BSM. It certainly takes a lot of time to calculate. Therefore, in this study, with limited steps, Jarrow-Rudd and Leisen-Reimer models are compared to BSM with the Cox-Ross-Rubinstein model. It aims to check on which binomial model is more convergent to BSM with limited steps in the same period. The data collected were secondary data from finance.yahoo.com. Judgemental sampling was used for technique sampling and several shares, with large market capitalization in Hongkong, India, and Indonesia are chosen. By calculating the MAFE error value from option price of BSM and BM, it is discovered that Leisen-Reimer with 101 steps is more convergent to BSM.

Keywords
black-scholes model; binomial model; cox-ross-rubinstein; jarrow-rudd; and leisen-reimer

I. Introduction

Stock option is one of the derivative products used as a financial risk management tool. It is a right owned by the *taker* to purchase *(call option)* or to sell *(put option)* to the *writer* on a number of shares *(underlying stock)* to price *(strike price)* at a particular time. Two option forms which are frequently used are American and European options. In the former, the option holder (taker) can execute their right all the time until the due date. Meanwhile, the latter only gives the opportunity to the taker to execute on the due date.

Since 1973, option exchange began its fast growth. In Asia, based on the data from The World Federation of Exchanges in 2020, Hongkong (HKEX) and India stock exchange (NSE) have become the largest ones according to the number of contracts of single stock options and entered the world's top ten in which the investors look up to.

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Table 1. Top 10 exchanges by number of single stock options contracts traded in 2020

	Volume		Notional Value		Open interest	
	2020	YoY change	2020	YoY change	2020	YoY change
Nasdaq - US	1,702,761,434	106%	NA	NA	NA	NA
B3 - Brasil Bolsa Balcão	1,459,181,587	48%	699,884	-19%	46,530,971	68%
Cboe Global Markets	1,287,889,878	57%	NA	NA	265,226,043	46%
NYSE	782,351,153	77%	362,895	175%	NA	NA
MIAX Exchange Group	507,985,576	95%	9,573,574	160%	NA	NA
International Securities Exchange	417,128,811	72%	NA	NA	NA	NA
National Stock Exchange of India	272,134,027	35%	2,719,053	60%	914,728	177%
Eurex	189,106,929	2%	929,810	4%	49,863,675	-2%
Hong Kong Exchanges and Clearing	129,601,933	22%	551,546	54%	8,860,221	34%
Euronext	69,478,712	2%	289,668	3%	12,519,273	12%
Others	221,570,604	-11%	557,880	35%	27,573,238	-5%
Grand Total	**7,039,190,644**	**60.5%**	**15,684,310**	**89%**	**411,488,149**	**34%**

In Indonesia, the first stock options trade commenced on October 6, 2004, with five main shares (Tandelilin, 2017). However, since 2010, the options market in the capital market in this country is no longer working (Dewi and Ramli, 2019). Even though the Indonesian market has not been working anymore and its Stock Exchange is not as large as Hongkong's and India's, Indonesia is considered to hold massive market potential.

The establishment of options prices is one of the pivotal aspects in derivative trade. One of its models is Black-Scholes Model (BSM). It was first introduced by Fischer Black and Myron Scholes in 1973 (Black and Scholes, 1973). In the same year, Robert Merton modified BSM to calculate dividend and interest rate variables. Up to these days, this model is still used as the reference for calculation option prices. In 1978, William F. Sharpe (Sharpe, 1978) proposed a new model of option price establishment for the first time called binomial model (BM). Then, in 1979, John C. Cox, Stephen Ross, and Mark Rubinstein (Cox, Ross, and Rubinstein, 1979) formulated the first completed binomial option price called Cox-Ross-Rubinstein (CRR). In 1982, Robert A. Harrow and Andrew Rudd (Jarrow and Rudd, 1983) formulated the Jarrow-Rudd model (JR). Next, in 1996, Dietmar Leisen and Matthias Reimer (Leisen and Reimer, 1998) introduced the Leisen-Reimer model (LR).

In 2012, Feng and Kwan (1983) investigated that eventually BM is convergent to BSM along with the increase of the Binomial period. Dar and Anuradha (2018) in their study, compare BM and BSM and discover that both models have quite close results in one periode.

In establishing option prices, most journals compare BSM with the binomial model of Cox-Ross-Rubinstein. Even though it is quite simple, to make CRR converge with BSM requires a lot, even thousands of steps. It certainly takes a lot of time to calculate. Thus, this paper does not only discuss the CRR model but also JR and LR. Those three binomial models are compared to BSM with limited steps. The number of steps used are the lower step or equal to 101. It aims to check with limited steps, which binomial model is more convergent with BSM in the same period. The data taken were from shared data from Hongkong, India, and Indonesia. The results of option price establishment from these three binomial models were compared with BSM. Then, MAFE error value (de Villiers, 2007) is gained from them. Binomial model which has the smallest MAFE is the most convergent one to BSM.

II. Research Methods

This study applied sampling judgemental technique and we chose several shares which have large market capitalization in Hongkong, India, and Indonesia. Hongkong shares used are from China Construction Bank Co. (CCB), China Merchants Bank Co., Ltd. (CMB), Industrial and Commercial Bank of China Ltd. (ICBC), and Tencent Holdings Ltd. (TH). The Indian ones are from HDFC Bank Ltd. (HDBK), Infosys Lrd. (INFY), Reliance Industries Limited (RELI), and Tata Consultancy Service Ltd. (TCS). In the meantime, the ones from Indonesia are from PT Astra International Tbk (ASII), PT Bank Central Asia Tbk (BBCA), PT Indofood Sukses Makmur Tbk (INDF), and PT Telekomunikasi Indonesia Tbk (TLKM). Those shares were selected as they have the largest market capitalization in each country. Market capitalization is the company measure of one country which indicates the current stock price multiplied by the circulating number of shares.

The historical data adopted from each country is distinguishable because every country has their own trading day. Stock historical data from Hongkong was taken on October 5, 2020 until April 12, 2021, India on October 6, 2020 until April 9, 2021, and Indonesia on September 29, 2020 until April 8, 2021. These daily historical data were collected from finance.yahoo.com.

Stock option is a European call started from January 7, 2021 for every share chosen with option period in one and three months. To calculate stock option prices, it requires inputs such as closing price, volatility, strike, dividend, and interest rate. The closing price used here was the adjusted closing price. It is because the adjusted closing price has been adjusted so that it gave an accurate illustration about the equity value of a company outside simple market price. Volatility could be calculated using adjusted closing price. One month option applied one month volatility (21 trading days) and three-month option used three-month historical volatility (63 trading days). Every company always tries to maximize the benefits it gets (Sitanggang *et al*, 2020). The strike price which was collected was to be positioned at the money/near at the money and adjusted according to the currency of each country. In addition, the dividend used was the average of the dividend in the last five years sourced from investing.com. Then, the interest rate used follows the Interbank Offered Rate of each country, namely HIBOR (hangseng.com), MIBOR (www.fimmda.org), and JIBOR (www.bi.go.id). The following table is the details of strike, dividend, and interest rate used.

Table 2. Strikes and dividends from each stock

Stock	Strike	Dividend
CCB	5.75	5.036%
CMB	52.5	2.788%
ICBC	4.7	5.050%
TH	580	0.210%
HDBK	1420	0.545%
INFY	1280	2.275%
RELI	1920	0.498%
TCS	3020	1.749%
ASII	6000	2.909%
BBCA	34500	1.177%
INDF	6800	3.353%
TLKM	3300	3.570%

Table 3. Interest rate on January 7, 2021

Interest Rate	HIBOR	MIBOR	JIBOR
1 month	0.09%	3.49%	3.80%
3 months	0.16%	3.57%	4.04308%

With those data, option prices were also calculated using two models namely BSM and BM, where BM was calculated using three models, namely Cox-Ross-Rubinstein, Jarrow-Rudd, and Leisen-Reimer. Next, BSM and BM were compared by applying the mean absolute forecast error (MAFE):

$$\text{MAFE} = \frac{1}{T}\sum_{i=1}^{T} |BSM_i - BM_i|$$

The shorter MAFE value is, the more convergent BSM and BM are.

2.1 Historical Volatility

Most derivative texts are instructed to apply historical volatility. It is a volatility that is observed on underlying asset prices in the past. Mathematically, it is a log return deviation standard from daily closing price.

The calculation is commenced by calculating log return gradually for each period. Log return can be calculated with the formula as follows:

$$R_i = ln\ ln\left(\frac{C_i}{C_{i-1}}\right)$$

ln = natural log

C_n = closing sequence (i)

C_{n-1} = closing price day sequence (i) $- 1$

The next one is calculating deviation standard from log return from the previous step. It needs to account that deviation standard is root square from the variants, which is the deviation square mean from the mean. Thus, the daily volatility formula is as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (R_i - R_{avg})^2}{n-1}}$$

R_i = log return sequence *(i)*

R_{avg} = log return mean

n = number of trading days

The last step is calculating the annual historical volatility by multiplying daily volatility with $\sqrt{252}$ (number of trading days in a year).

2.2 Black-Scholes Model (BSM)

This model was first introduced by Fischer Black and Myron Scholes in *Journal of Political Economy* to calculate the price of an option (Black and Scholes, 1973). This paper has high popularity and becomes the reference of all parties until these days in discussing the price of an option. Scholes together with Robert Merton (Merton, 1973) (who independently also developed the derivative assessment formula) developed that model becoming a new one, which is known as Black-Scholes-Merton model and won the Nobel Prize in economy in 1997.

In BSM, option prices of call and put with dividend can be calculated with the following formula.

$$C = S_0 e^{-qt} * N(d_1) - K e^{-rt} * N(d_2)$$
$$P = K e^{-rt} * N(-d_2) - S_0 e^{-qt} * N(-d_1)$$

with

$$d_1 = \frac{ln\ ln\ \left(\frac{S_0}{K}\right) + t\left(r - q + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{t}}$$
$$d_2 = d_1 - \sigma\sqrt{t}$$

Notes:

S_0 = underlying price

K = strike price

σ = volatility (%)

r = interest rate (%)

q = dividend (%)

t = time to expiration (% of year)

Example 1 European call options in ASII stock with $S_0 = 5978.905$, $K = 6000$, $\sigma = 31.56\%$, $r = 3.80\%$, $q = 2.91\%$, dan $t = 7.67\%$ (1 month). Option price is calculated with the following BSM formula:

$$d_1 = \frac{ln\ ln\ \left(\frac{5978.905}{6000}\right) + 7.67\%\left(3.80\% - 2.91\% + \frac{31.56\%^2}{2}\right)}{31.56\%\sqrt{7.67\%}}$$
$$= \frac{-0.0035 + 0.0045}{0.0874}$$
$$= 0.0112$$
$$d_2 = 0.0112 - 0.0874$$
$$= -0.0762$$
$$C = 5978.905_0 e^{-(2.91\%)(7.67\%)} * N(0.0112) - 6000 e^{-(3.80\%)(7.67\%)} * N(-0.0762)$$
$$= 199.879$$

Therefore, the Europe call option obtained is 199.879.

2.3 Binomial Model

It was first proposed by William F. Sharpe (Sharpe, 1978). Then in 1979, John C. Cox, Stephen Ross, and Mark Rubinstein formulated the establishment of the first completed binomial option price (Cox, Ross, and Rubinstein, 1979). In this model, the option price is calculated by dividing the expiration time into several steps and simulating price movement with a binomial tree. Although computationally this model is slower than Black-Scholes one, it is more accurate, particularly for long-term options with dividend.

a. Cox-Ross-Rubinstein Model

It is the first completed binomial option price which is quite prominent and the simplest one to calculate. The formula of up-move and down-move measure is as follows:

$$u = e^{\sigma\sqrt{\Delta t}}$$

$$d = \frac{1}{u}$$

The probability formula of up-move with dividend result is as follows:

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

Δt value is the quotient of days to expiration with the number of steps, while the probability formula of down-move is 1 – *p*, as up-move and down-move must be as many as one.

b. Jarrow-Rudd Model

This model was first introduced by Robert A. Jarrow and Andrew Rudd in 1983 (Jarrow and Rudd, 1983). It remarks that the resulting option price is stated as the sum of three Black-Scholes price plus the adjustment depending on the stochastic process *security*. Similar to the other binomial option price establishment, Jarrow-Rudd binomial trees are determined by the size and probability of up-move and down-move. The primary characteristic of it is that up-move and down-move have the same probability, 50% for each. It is occasionally mentioned as the same probability model.

$$p = 0{,}5$$

The formula of up-move and down-move size is as follows.

$$u = e^{(r-q-\frac{\sigma^2}{2})\Delta t + \sigma\sqrt{\Delta t}}$$

$$d = e^{(r-q-\frac{\sigma^2}{2})\Delta t - \sigma\sqrt{\Delta t}}$$

c. Leisen-Reimer Model

This model was introduced by Dietmar Leisen and Matthias Reimer (Leisen and Reimer, 1996). One thing that must be accounted for is that the number of steps must be odd. If it is not odd, it can cause the option price to be incorrect. The probability must be calculated before movement size. The calculation requires d_1 and d_2 from Black-Scholes model.

The function of $h^{-1}(z)$ is Peizer-Pratt inverse function, which gives the binomial estimation (discreet) to normal cumulative distribution function (continual). The formula of Peizer-Patt inverse function is as follows.

$$h^{-1}(z) = \frac{1}{2} + \frac{sign(z)}{2}\sqrt{1 - e^{\left[-\left(\frac{z}{n+\frac{1}{3}+\frac{0.1}{n+1}}\right)^2\left(n+\frac{1}{6}\right)\right]}}$$

When using this formula to calculate $p = h^{-1}(d_2)$, use d_2 in z and n is the number of steps from the binomial model.

The formula for the Leisen-Reimer up-move and down-move is as follows:

$$u = e^{(r-q)\Delta t} \,.\, \frac{p'}{p}$$

$$d = e^{(r-q)\Delta t} \,.\, \frac{1-p'}{1-p}$$

with

$$p' = h^{-1}(d_1)$$

There are three pivotal points which must be taken into account when calculating option price with binomial model:

1) Binomial price tree
   It is based on the underlying price to calculate stock price until expiration day. Each step is assumed that the underlying price moves either up or down. Up-move, down-move, and probability are calculated with one of the models, that is Cox-Ross-Rubinstein, Jarrow-Rudd, or Leisen-Reimer.
2) Calculation of the option price at each final node
   At each final node from the price tree (at the end of the option), the price option is only the intrinsic value. The up-move and down-move of price option in the expiration period for *call* and *put* options are as follows:
   $C = maximum(0, S - K)$
   $P = maximum(0, K - S)$
   There are two probabilities when expiring. First, if the option ends with profit potential, *exercise* can be done, and it gains the difference between underlying price S and the deal price K. From *call*, $S - K$ is obtained. From one put, $K - S$ is obtained. Second, if the option ends with disadvantage potential, *exercise* can be not chosen and let the last option end. Option price is zero in that case.
3) Sequent calculation of option price at each previous point
   The converse applies in this option tree from the price tree. It is calculated from right to left using the two nodes behind. By understanding intrinsic value (O), option price is calculated as follows.
   $E = \left(O_u . p + O_d . (1 - p)\right) . e^{-r\Delta t}$
   With this formula, option price can be calculated in all nodes starting from expiration until the first node of the tree. This first node is called price option.
   Further explanation about binomial trees can be read in Trenca and Pochea, 2010.

Example 2 European call option on ASII stock starting on January 7, 2021, with $S_0 = 5978.905$, $K = 6000$, $\sigma = 31.56\%$, $r = 3.80\%$, $q = 2.91\%$, $t = 7.67\%$ (1 month), and step $= 20$**.** The values of up-move and down-move are calculated with CRR binomial model formula as follows:

$$u = e^{31.56\% \sqrt{\frac{7.67\%}{20}}} = 1.01973674$$
$$d = \frac{1}{1.01973674} = 0.98064526$$

Underlying price on step 1:
$S_{1u} = S_0 . u = (5978.905)(1.01973674) = 6096.909$
$S_{1d} = S_0 . d = (5978.905)(0.98064526) = 5863.185$

The underlying price on the next step can be obtained with similar method by multiplying it from the previous step with u when moving up and with d when moving down. Hence, the following underlying tree is obtained.

Time	0.00%	0.38%	0.77%	1.15%	1.53%	1.92%	2.30%	2.68%	3.07%	3.45%	3.84%	4.22%	4.60%	4.99%	5.37%	5.75%	6.14%	6.52%	6.90%	7.29%	7.67%
Step	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	5978.905	6096.909	6217.242	6339.95	6465.08	6592.68	6722.798	6855.484	6990.788	7128.764	7269.462	7412.938	7559.245	7708.44	7860.579	8015.722	8173.926	8335.252	8499.763	8667.521	8838.589
1		5863.185	5978.905	6096.909	6217.242	6339.95	6465.08	6592.68	6722.798	6855.484	6990.788	7128.764	7269.462	7412.938	7559.245	7708.44	7860.579	8015.722	8173.926	8335.252	8499.763
2			5749.704	5863.185	5978.905	6096.909	6217.242	6339.95	6465.08	6592.68	6722.798	6855.484	6990.788	7128.764	7269.462	7412.938	7559.245	7708.44	7860.579	8015.722	8173.926
3				5638.42	5749.704	5863.185	5978.905	6096.909	6217.242	6339.95	6465.08	6592.68	6722.798	6855.484	6990.788	7128.764	7269.462	7412.938	7559.245	7708.44	7860.579
4					5529.29	5638.42	5749.704	5863.185	5978.905	6096.909	6217.242	6339.95	6465.08	6592.68	6722.798	6855.484	6990.788	7128.764	7269.462	7412.938	7559.245
5						5422.272	5529.29	5638.42	5749.704	5863.185	5978.905	6096.909	6217.242	6339.95	6465.08	6592.68	6722.798	6855.484	6990.788	7128.764	7269.462
6							5317.325	5422.272	5529.29	5638.42	5749.704	5863.185	5978.905	6096.909	6217.242	6339.95	6465.08	6592.68	6722.798	6855.484	6990.788
7								5214.41	5317.325	5422.272	5529.29	5638.42	5749.704	5863.185	5978.905	6096.909	6217.242	6339.95	6465.08	6592.68	6722.798
8									5113.486	5214.41	5317.325	5422.272	5529.29	5638.42	5749.704	5863.185	5978.905	6096.909	6217.242	6339.95	6465.08
9										5014.516	5113.486	5214.41	5317.325	5422.272	5529.29	5638.42	5749.704	5863.185	5978.905	6096.909	6217.242
10											4917.462	5014.516	5113.486	5214.41	5317.325	5422.272	5529.29	5638.42	5749.704	5863.185	5978.905
11												4822.285	4917.462	5014.516	5113.486	5214.41	5317.325	5422.272	5529.29	5638.42	5749.704
12													4728.951	4822.285	4917.462	5014.516	5113.486	5214.41	5317.325	5422.272	5529.29
13														4637.424	4728.951	4822.285	4917.462	5014.516	5113.486	5214.41	5317.325
14															4547.668	4637.424	4728.951	4822.285	4917.462	5014.516	5113.486
15																4459.649	4547.668	4637.424	4728.951	4822.285	4917.462
16																	4373.333	4459.649	4547.668	4637.424	4728.951
17																		4288.689	4373.333	4459.649	4547.668
18																			4205.682	4288.689	4373.333
19																				4124.282	4205.682
20																					4044.458

Figure 1. *Binomial underlying tree*

Then, the probability is calculated as follows:

$$p = \frac{e^{(3.80\% - 2.91\%)\frac{7.67\%}{20}} - 0.98064526}{1.01973674 - 0.98064526} = 0.49598829$$

At the option price tree, the calculation starts from right to left, hence the search focuses on the option price when first expiring. As it is a call option, then when underlying is below strike price, option will not be exercised and the value is 0. When underlying is bigger than strike price, then the option value is $S - K$. For instance, when 2838.589 on step 20 in Figure 2 is obtained from the subtraction of underlying price on step 20 in Figure 1 with strike price, then 8838.589-6000=2838.589. Next, to find out the price on step 19 is by applying the following formula:

$$E = \left(2838.589 \times 0.49598829 + 2499.763(1 - 0.49598829)\right).e^{-(3.80\%)\frac{7.67\%}{20}}$$
$$= 2667.428$$

This calculation was done until step 1 and the option price gained was 198.9766.

Time	0.00%	0.38%	0.77%	1.15%	1.53%	1.92%	2.30%	2.68%	3.07%	3.45%	3.84%	4.22%	4.60%	4.99%	5.37%	5.75%	6.14%	6.52%	6.90%	7.29%	7.67%
Step	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	198.9766	238.3137	329.7363	413.6758	509.9349	617.6408	735.3254	861.5441	993.2004	1129.886	1270.117	1413.363	1559.493	1708.54	1860.563	2015.622	2173.775	2335.086	2499.613	2667.428	2838.589
1		140.6415	188.1028	247.2284	319.0688	404.091	502.0085	611.7224	731.439	858.9775	992.2151	1129.518	1269.968	1413.269	1559.431	1708.512	1860.369	2015.662	2173.851	2335.197	2499.763
2			93.97647	129.9729	176.6032	235.4924	307.8495	394.1859	494.0884	606.1425	728.1093	857.3845	991.6309	1129.337	1269.842	1413.174	1559.369	1708.483	1860.574	2015.702	2173.926
3				58.58029	84.12244	118.7025	164.3554	222.9761	295.9878	383.9611	486.2925	601.1026	725.5231	856.4239	991.3548	1129.159	1269.716	1413.08	1559.307	1708.454	1860.579
4					33.46168	50.11718	73.80072	106.7154	151.1912	209.5004	283.3698	373.4508	478.8365	596.9161	723.8886	856.0311	991.1662	1129.001	1269.589	1412.985	1559.245
5						17.085	26.80532	41.45117	62.97848	93.85401	136.8676	194.8047	269.8508	362.7752	472.1374	594.0592	723.2948	855.8122	990.9774	1128.843	1269.462
6							7.506589	12.42021	20.27854	32.61269	51.5523	79.89235	121.0097	178.4837	255.2588	352.2931	467.0527	593.0961	723.0462	855.5932	990.7885
7								2.673358	4.690573	8.146605	13.98401	23.6783	39.45269	64.48555	102.9825	159.8429	239.4623	343.1509	465.3862	592.8184	722.7975
8									0.689029	1.290915	2.404484	4.448087	8.161871	14.82974	26.62005	47.05709	81.53726	137.4936	222.9607	340.1171	465.0799
9										0.096924	0.195445	0.394109	0.79471	1.602509	3.231413	6.516048	13.13942	26.49525	53.42689	107.7338	217.2419
10											0	0	0	0	0	0	0	0	0	0	0
11												0	0	0	0	0	0	0	0	0	0
12													0	0	0	0	0	0	0	0	0
13														0	0	0	0	0	0	0	0
14															0	0	0	0	0	0	0
15																0	0	0	0	0	0
16																	0	0	0	0	0
17																		0	0	0	0
18																			0	0	0
19																				0	0
20																					0

Figure 2. *Binomial Option Price Tree*

III. Results and Discussion

3.1 MAFE comparison for one month option

European call option has been observed with a one-month option period (21 trading days) started from January 7, 2021. With the determined volatility, strike, interest rate, and dividend for each share, thus option price for BSM and BM model can be calculated with excel assistance. Binomial models of Cox-Ross-Rubinstein and Jarrow-Rudd are calculated for step 20, 40, 60, 80, and 100, while Leisen-Reimer one is calculated for step 21, 41, 61, 81, and 101. Those results are then compared, then it will obtain the following MAFE results.

Table 4. The comparison of MAFE of Cox-Ross-Rubinstein and BSM for one month option

Binomial Model	CRR 20	CRR 40	CRR 60	CRR 80	CRR 100
CCB	0.09849%	0.04965%	0.02133%	0.02159%	0.01561%
CMB	1.02387%	0.66782%	0.28565%	0.27245%	0.24018%
ICBC	0.09496%	0.04006%	0.03102%	0.02401%	0.01763%
TH	13.77742%	8.23106%	5.18046%	3.24929%	3.61426%
HDBK	15.94731%	10.96431%	7.20174%	5.35120%	4.84117%
INFY	27.58209%	13.51762%	10.18837%	6.53098%	5.03776%
RELI	44.40812%	23.01880%	14.01831%	9.24717%	7.79149%
TCS	39.15947%	18.87700%	13.42444%	11.67773%	9.77693%
ASII	139.47649%	73.90027%	44.00915%	32.18619%	26.88896%
BBCA	753.51977%	396.91591%	236.07992%	214.36040%	142.11318%
INDF	96.09280%	55.94880%	33.89075%	24.60423%	22.99856%
TLKM	110.86741%	45.28170%	26.04320%	23.66919%	19.56091%

Table 4 displays the MAFE calculation results from the option price CRR and BSM model. It indicates that the smallest MAFE is in the CBB stock with CRR on step 100 by 0.01561%. On average, MAFE from step 20 to 100 is decreasing. It points out that as the CRR model step increases, the more convergent it is with BSM.

Table 5. The comparison of MAFE of Jarrow-Rudd and BSM for one month option

Binomial Model	JR 20	JR 40	JR 60	JR 80	JR 100
CCB	0.09015%	0.05020%	0.02421%	0.02331%	0.01804%
CMB	1.07682%	0.53204%	0.41196%	0.34134%	0.28224%
ICBC	0.09616%	0.04157%	0.02722%	0.02076%	0.01811%
TH	15.07237%	6.81709%	4.63852%	3.15943%	2.92029%
HDBK	15.64636%	10.18863%	7.07773%	5.33106%	3.53351%
INFY	25.91031%	11.28201%	8.66782%	7.23890%	5.59289%
RELI	39.30991%	20.57407%	11.81695%	7.62260%	8.13765%
TCS	39.42236%	20.65761%	16.26301%	10.73222%	10.11543%
ASII	131.00451%	48.67471%	50.46151%	26.77111%	24.95671%
BBCA	756.01978%	395.04714%	263.90090%	203.75061%	147.29903%
INDF	85.43228%	47.79139%	33.43600%	24.50742%	21.81330%
TLKM	116.07810%	48.94367%	31.39025%	15.91529%	15.09202%

Table 5 indicates that the smallest MAFE is on CBB stock with JR step 100 by 0.01804%. On average, MAFE from step 20 to 100 is decreasing. It denotes that as the JR model step increases, the more convergent it is to BSM.

Table 6. The comparison of MAFE of Leisen-Reimer and BSM for one month option

Binomial Model	LR 21	LR 41	LR 61	LR 81	LR 101
CCB	0.00091%	0.00025%	0.00011%	0.00006%	0.00004%
CMB	0.00802%	0.00216%	0.00099%	0.00056%	0.00036%
ICBC	0.00132%	0.00036%	0.00016%	0.00009%	0.00006%
TH	0.18606%	0.04996%	0.02274%	0.01295%	0.00835%
HDBK	0.18125%	0.04917%	0.02247%	0.01282%	0.00828%
INFY	0.29857%	0.08102%	0.03704%	0.02114%	0.01364%
RELI	0.51330%	0.13935%	0.06372%	0.03636%	0.02348%
TCS	0.36858%	0.09983%	0.04561%	0.02602%	0.01680%
ASII	1.40029%	0.37922%	0.17324%	0.09881%	0.06377%
BBCA	9.87818%	2.68351%	1.22743%	0.70056%	0.45232%
INDF	1.19716%	0.32413%	0.14806%	0.08444%	0.05450%
TLKM	1.29847%	0.35282%	0.16143%	0.09215%	0.05950%

From Table 6, it can bee seen that the smallest MAFE is on CBB stock with LR step 101 by 0.00004%. On average, MAFE from step 20 to 100 is decreasing. It points out that as the JR mode step increases, the more convergent it is to BSM.

Based on those results, it is discovered that MAFE from BSM and BM option prices are decreasing as step increases. With step ≤ 101Cox-Ross-Rubinstein and Jarrow-Rudd models have almost similar results and it can be determined which one is more convergent to BSM due to their volatile differences. Meanwhile, for the MAFE of Leisen-Reimer, it is much smaller than the previous two models. For step $\leq 101,$ This model with 101 steps has a much smaller MAFE value for all shares. Thus, it can be determined that for a one-month option, with limited steps, Leisen-Reimer with 101 steps is more convergent to BSM than the other ones.

3.2 MAFE Comparison for three-month option

We observed European call options, with three-month options (63 trading days) starting from January 7, 2021. With the determined volatility, strike, interest rate, and dividend for each share, the price option for BSM and BM model can be calculated through Excel. Binomial models of Cox-Ross-Rubinstein and Jarrow-Rudd are calculated for step 21, 41, 61. 81, and 101 and those results will be compared later. The MAFE values obtained are as follows.

Table 7. The comparison of MAFE of Cox-Ross-Rubinstein and BSM for three-month option

Binomial Model	CRR 20	CRR 40	CRR 60	CRR 80	CRR 100
CCB	0.17733%	0.10127%	0.06224%	0.05480%	0.04244%
CMB	36.03921%	18.40028%	10.96688%	7.76481%	7.20537%
ICBC	0.18587%	0.08584%	0.05873%	0.04380%	0.03344%
TH	36.03921%	18.40028%	10.96688%	7.76481%	7.20537%
HDBK	57.63812%	24.49256%	14.93414%	13.63851%	11.14728%
INFY	41.73916%	20.85332%	15.47581%	11.72219%	7.58015%
RELI	78.87806%	38.07593%	26.53427%	18.45720%	15.89912%
TCS	92.04593%	45.91965%	27.51969%	22.71593%	19.31273%
ASII	252.81240%	128.75459%	88.30287%	68.48842%	53.25188%
BBCA	1226.72759%	604.15731%	395.67792%	309.08363%	241.86405%
INDF	175.04571%	97.57278%	71.22531%	45.72501%	36.37580%
TLKM	189.50396%	97.14966%	64.54149%	47.65302%	34.58463%

From Table 7, it can be concluded that the smallest MAFE exists on CBB shares with step 100 CRR by 0.04244%. On average, MAFE is decreasing from step 20 to 100, showing that as CRR model step increases, the more convergent it is to BSM.

Table 8. The comparison of MAFE of Jarrow-Rudd and BSM for three-month option

Binomial Model	JR 20	JR 40	JR 60	JR 80	JR 100
CCB	0.20252%	0.09350%	0.06363%	0.04385%	0.03716%
CMB	34.09181%	18.29215%	11.63137%	8.45671%	7.08236%
ICBC	0.17420%	0.09983%	0.06319%	0.04593%	0.04148%
TH	34.09181%	18.29215%	11.63137%	8.45671%	7.08236%
HDBK	57.10779%	25.76684%	16.38330%	14.18958%	11.40293%
INFY	38.74143%	23.00726%	15.77956%	12.19931%	9.31021%
RELI	78.58080%	41.15807%	26.63036%	18.99762%	14.92922%
TCS	105.07916%	47.92887%	29.13284%	21.81608%	18.66426%
ASII	253.95782%	135.55756%	86.07800%	65.57241%	53.74762%
BBCA	1212.25445%	590.96834%	395.53664%	327.44442%	247.27167%
INDF	183.21001%	86.34055%	56.48415%	45.31394%	36.12936%
TLKM	180.23097%	92.79763%	57.36941%	47.85205%	38.36164%

Table displays that the smallest MAFE is on CBB share with step 100 JR by 0.01804%. On average, it is lowering from step 20 to 100, denoting that as the JR model step increases, the more convergent it is to BSM.

Table 9. The comparison of MAFE of Leisen-Reimer and BSM for three-month option

Binomial Model	LR 21	LR 41	LR 61	LR 81	LR 101
CCB	0.00252%	0.00068%	0.00031%	0.00018%	0.00012%
CMB	0.32296%	0.08770%	0.04011%	0.02289%	0.01478%
ICBC	0.00225%	0.00061%	0.00028%	0.00016%	0.00010%
TH	0.32296%	0.08770%	0.04011%	0.02289%	0.01478%
HDBK	0.55567%	0.15136%	0.06931%	0.03958%	0.02556%
INFY	0.52199%	0.14191%	0.06493%	0.03706%	0.02393%
RELI	0.85103%	0.23137%	0.10587%	0.06044%	0.03903%
TCS	1.12781%	0.30680%	0.14041%	0.08016%	0.05177%
ASII	2.83429%	0.77154%	0.35318%	0.20166%	0.13024%
BBCA	14.97410%	4.06751%	1.86040%	1.06181%	0.68556%
INDF	1.88893%	0.51299%	0.23461%	0.13390%	0.08645%
TLKM	2.61559%	0.71026%	0.32482%	0.18537%	0.11968%

From Table 9 above, it can be viewed that the smallest MAFE exists on CBB shares with step 101 LR by 0.00004%. On average, it is decreasing from step 20 to 101, pointing out that as the JR model step increases, the more convergent it is to BSM.

Based on those results, as the step increases, MAFE from the price options of BSM and BM is decreasing. With step ≤ 101The models of Cox-Ross-Rubinstein and Jarrow-Rudd have almost similar results in which it cannot be determined which one is more convergent to BSM as the difference is volatile. Meanwhile, for Leisen-Reimer's MAFE, it is much smaller than the previous two. For step ≤ 101, Leisen-Reimer model with 101 steps has the smallest MAFE value for all shares. Thus, it can be determined that for a three-month option, with limited steps, Leisen-Reimer with 101 steps is more convergent to BSM compared to the other ones.

IV. Conclusion

The establishment of option prices can be achieved with BSM and BM models. The procedures applied denote that as the step in binomial options increases, the more convergent the binomial option price is to BSM. With one month or three-month option and down-move binomial step or equal to 101, hence Leisen-Reimer with 101 steps is more convergent to BSM.

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