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# Development of Mathematics Learning Model Concept-Based Understanding

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## Abstract

*This research is the development of a mathematical learning model based on conceptual understanding. The main research problem: How to develop students' current understanding which tends to turn from factual and procedural knowledge to conceptual knowledge? The purpose of the research is to build and develop the emergence of students' conceptual knowledge, as well as to identify and analyze the lecturer's conceptual knowledge. Data were collected through direct observation in the classroom while learning was taking place in the Mathematics Education Study Program. Observations in three different classes, namely Algebra Class, Calculus Class, and Geometry Class. The display and class discussion were recorded by the researcher, especially on some of the problems that arose. Then the researcher developed a discussion through conceptual questions, which aimed to build conceptual knowledge and simultaneously develop prior knowledge from educators and students. The data recording mechanism is from the classroom so that in the first stage, researchers get a class model in learning or teaching mathematics. From the initial model, the researcher and the class developed conceptual thinking of the problem or problem solving as a form of testing the validity of the model. Testing is carried out primarily to avoid factual and procedural knowledge. The results of this study are the identification of mathematical knowledge in learning and can be known how to work and the way the class thinks as a whole. Overall, an analysis of the concept-based learning model is generated.*

Keywords
factual knowledge; procedural knowledge; conceptual knowledge; mathematics learning; math learning model

## I. Introduction

Until now, mathematics learning still emphasizes the presentation of factual and procedural knowledge (Rif'at, 2009). In fact, the core standard in learning mathematics emphasizes concept understanding as a key component of mathematical proficiency. Too many think that if students know all the definitions and rules, then they have a conceptual understanding. The standard arguably offers very few mathematical learning models for educators. The limitations of the model can be recognized, among others, is that "understanding" means justifying the procedures used or stating the reasons for a process. Mathematics learning presented using Student Worksheets requires active participation from students, because the Student Worksheets are a form of teacher effort to guide students structured through activities that are able to attract students to learn mathematics. In addition, learning with Student Worksheets can make the learning process more effective as expected in each learning that is increasing the creativity of students' thinking so that learning objectives are achieved. (Tarigan, E. et al. 2020)



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The mark of the quality of mathematical understanding is the ability to reason, in a way that is in accordance with the mathematical perfection of the learner. For example, why a mathematical statement is true or where a mathematical rule appears. The mark of quality should be able to vary on the basis of differences between students. Research on conceptual understanding in mathematics found that: "successful learning of mathematics requires three different abilities that must be developed and assembled together, namely: fact control, process control, and conceptual understanding". However, of the three types of knowledge that students need, conceptual understanding is the most difficult to convey. One reason is that conceptual knowledge is often not readily available anymore and educators cannot transfer it directly to the learner's 'head'.

Another consequence of the failure to transfer conceptual knowledge is a shift in emphasis from learning to an emphasis on learning mathematics. In this case, the emphasis on learning is experiencing the processes and procedures of the mathematical discipline. The researcher's point of view is on the determination of educators in expanding or developing practical work so far, and not carrying out fact-based learning, properties, laws, principles and theories that cover the content of mathematical knowledge accompanied by the use of discovery and inquiry methods in learning.

The implementation of learning that 'covers' the content of mathematical knowledge is an example of a learning paradox where the educator as the center of knowledge is neglected. This model is called direct learning. In the context of meaning, direct learning needs to be carried out in cognitive strategies. Direct learning with cognitive strategies is useful in literacy. The reason is that educators cannot directly teach understanding the meaning of reading that goes beyond concepts as facts.

The core of this research is the argument of setting learning objectives in mathematics. So far, the learning objectives are directed at achieving knowledge and skills in solving routine problems and which accommodate students to find key facts and skills for themselves. The argument is 'narrow' and does not follow all learning events that occur through direct learning where knowledge equals understanding.

While the learning objectives in this study put the importance of mastering conceptual knowledge. In the cognitive aspect, one of the things that must be mastered in learning mathematics is understanding. According to Bloom (in Anderson, 2001), understanding is the ability to capture the meaning of the material being studied. Not always someone (student) can learn concepts directly from the definition, for example for those who are not yet at the level of formal thinking. According to Skemp (1987), concepts that are higher order than a person's concept can not always be communicated well to the person through a definition, but it is necessary to first provide him with a set of examples of the concept. Ausubel (1968) suggests that concepts can be obtained in two ways, namely concept formation and concept assimilation. Concept formation can be viewed as learning concrete concepts, whereas concept assimilation is relevant to learning abstract concepts.

Concept formation is an inductive process. In this process a person abstracts certain attributes that are the same from various given stimuli. While the assimilation of concepts is deductive. In this process a person learns a concept based on the introduction of the term or name of the concept and its attributes. Therefore, conceptual learning includes formulating concepts and determining what is obtained, then formulating new properties, in the form of principles or theorems.

## II. Review of Literature

### 2.1 Research Roadmap

Learning is carried out through questions. Starting from giving problems that contain the main idea that supports the formation of conceptual understanding in stages. The whole stage includes three (3) stages.

The first stage, research respondents develop conceptual knowledge from procedural and conceptual knowledge. In general, in the first stage, the researcher begins the discussion by accepting procedural rules in order to lead to conceptual knowledge. This stage culminates in an analysis of the research subject's work. Researchers accommodate to carry out activities that promote the use of student choice and discovery strategies to solve problems.

### 2.2 Reference Library

The results of research in the field of cognitive psychology are described as a type of cognitive change of students so that they can succeed in learning mathematics (Nunes, 1992). Cognitive strategies refer to specific strategies that students can use as support in initiating learning activities. That researchers do not fully recognize that they can capture all aspects of expertise, competence, knowledge, and facilitation in mathematics, but choose mathematical skills to capture conceptual knowledge that arises in learning mathematics and succeeds.

One component of mathematical proficiency is conceptual understanding. Conceptual understanding includes understanding mathematical concepts, operations, and relations. Conceptual understanding provides a framework for discussion of the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency. The framework (Campbell, Voelkl, & Donahue, 2000) is similar to that used in mathematics assessment by the National Assessment of Educational Progress (NAEP), which characterizes the three mathematical abilities (conceptual understanding, procedural knowledge, and problem solving) and contains additional specifications. for reasoning, connection, and communication.

## III. Research Method

### 3.1 Theoretical Approach and Outcome

The theoretical approach of this research is constructivist. The theoretical approach is an attempt to apply constructivist theory with a change in emphasis, namely from different learning from the discipline of mathematics to the emphasis on learning mathematics through the processes and procedures of the discipline (Donovan, Bransford, & Pellegrino. (Eds.), 1999). The approach is carried out based on research where the "discovery" approach in learning so far will be tested regarding: (1) The researcher insists that the discovery or problem-based learning approach or project-based learning, all of which together do not help in the context of the cognitive meaning of "constructivism" with constructivist learning. The meaning should be related to the psychological theory of how the mind understands concepts. Researchers are of the opinion as believed by Schoenfeld (1989), that the approach referred to has not proven the theory of how to learn best; (2) The researcher asserts that the approach used is a theoretical and empirical review of inductive pedagogy so that it is in accordance with the needs of students and the treatment given. The position of the researcher in this study is in line with that of Maher & Martino (1996).

The research outputs, which focus on the assumption of working together with students, lecturers and mathematicians, through experience, mainly based on learning procedures and mathematical disciplines, are: (1) Expansion of practical work and projects and (2) Strengthening of learning with greater emphasis on application skills practical application of inquiry and problem solving as well as theoretical coverage in the discipline of mathematics accompanied by discovery and inquiry learning methods.

This study traces the emergence of conceptual understanding or knowledge from educators and students in the mathematics education study program FKIP Untan 2017. The search is carried out directly in the classroom during lectures. Researchers record data on the emergence of concepts distinguished from empirical characteristics and capture the reasons for getting the accuracy or truth of a concept that appears.

During the search and recording of data, the researcher focused on the principle of knowledge and which was defined as the level of affirmation of reality. The goal is that the function of intelligence is to encourage adaptation. The goal is related to the level of argumentation beyond the truth of a concept, both subjective and objective.

## IV. Result and Discussion

The observed mathematics educators raise requests to students to explain the process of procedures in mathematical work. There is still the ability to memorize formulas or mathematical principles or concepts, and something 'new' cannot be recorded yet. Presentation of concepts by educators is a simplification to understanding abstract ideas, including iconic presentations.

The appearance of information representation in the form of images, which is richer and is a type of active learning, does not always produce new concepts. Symbolic representation is better known to students so that conceptual knowledge is difficult to emerge. From the point of view of educational psychology, these learning events are critical. In this case, the epistemology of mathematics education is useful in extending help to find ways for students with early and incorrect ideas so that they understand scientific frameworks and mathematical concepts.

Another common conceptual problem is the understanding of the location of the image (geometry) which is always according to common or ordinary perceptions. This perception is a misconception. The essential properties have not been observed by students. For example, the shape of a gem (diamond or diamond) is a square or a rhombus.

The general problem of conceptual understanding as intended can be solved using an approach or reading strategy instead of each concept. For example, the misconception is that the length of the side of a square is equal to the length of its diagonal. Strategies to build understanding of the concept can start from the diagonal and find the sides of the square so that it can be compared correctly.

### 4.1 Findings

Indeed, in general the conceptual understanding of educators and students is still relatively lacking, even they do not understand it. This is based on the idea that if they know all the definitions and rules, then they have a conceptual understanding. From exploration, it is known that such educator thinking is a little offer for educators who must continue to develop.

Teaching materials that are displayed or written by educators rarely write down understanding in the form of justifying the procedures used or stating reasons in the process of working on problems. Related to this, it was found that the main foundation of the learning process is remembering and memorizing. Educators also repeat previous rote knowledge so that students are stimulated. How about understanding? There is hardly any in-depth exploration to build conceptual understanding.

This study also found the conceptual understanding of students who were assumed to have passed traditional mathematics courses in algebra and geometry. Namely, the lecturer asks students to explain the answers obtained from the problem of dividing zero. In general, all students stated that it could not be done. They state not defined or equal to infinity. However, the conceptual understanding of the division can actually be explored by defining the meaning of the "=" sign.

An interesting finding recorded during the lesson was that memory or memory still emerged that the multiplication of two numbers with a negative 'sign' resulted in a positive number. Such memories are not pursued by educators in order to build an understanding of related concepts. Similarly, geometric representations, such as the use of number lines or other geometric objects. It was found that the objects in the geometric representation were processed in the algebraic representation. It turns out that changing the representation eliminates the understanding of the concept. For example, the movement of points is known as an algebraic operation.

An important finding during the research was in the context of the primacy of mathematical work. Most, if not all, of mathematical work only produces facts, either because of definitions or rules or in finding answers. Applications or examples of getting the limit value of a particular function using the actual procedure only to produce facts. The learning activity that stands out is in applying procedural knowledge where the work as understood has been completed.

However, from procedural work there has not been a conceptual understanding in it. What does conceptual understanding look like? One sign is the ability to reason, in a way that is appropriate to the student's mathematical maturity. For example, certain mathematical statements are true or where the origin of a mathematical rule. There is a difference between students who can come up with a mnemonic device in describing the product of $(a + b)(x + y)$ and students who can explain the origin of the mnemonic.

Some of the "understanding" standards provide further understanding, namely: Students understand the connections between concepts. Understanding these connections includes applying the properties of a concept and creating and using strategies based on these traits to solve a problem. An additional emphasis on understanding connections is comparing various problem-solving strategies, so that students build their understanding of the relationships between concepts.

Understanding requires focusing the work of drawing conclusions. The work helps generalize from specific knowledge that is key to general understanding. The work is the virtue of conceptual understanding in mathematics along with mastery of two pillars namely factual knowledge and procedural skills. The finding of this research is that students learn routine problems using certain procedures. Meanwhile, conceptual knowledge or understanding of meaning has not yet emerged. For example, there is still knowledge that the product of two negative numbers produces a positive number, but it does not arrive at why this principle is true.

Indeed, procedural knowledge is no guarantee of conceptual understanding. It was found that the procedure for performing the operation was without an understanding of why the procedure worked. Observers of this study agree that procedural and conceptual knowledge is absolutely necessary. The researcher found that the results were very lacking in content and procedural knowledge, as revealed in the test. The researcher also notes that the main error is failing in conceptual understanding. The study conducted found that there were still many students who did not fully understand the concept. The mathematics education study program has not paid attention to conceptual understanding, for example, rigidly displaying these aspects of competence. For example, researchers still find arguments that 0.015 is greater than 0.05 because "15 is greater than 5."

### 4.2 Discussion

The type of mathematical knowledge that relates to facts is still prominent, such as 3/0 = . Indeed, such a fact raises problems. For example, the class cannot do mathematical work, it is difficult to visualize, and there is no conceptual understanding of why division by zero can be done.

The discussion about the number made is related to the rules for finding the intersection of cubic and line functions. Each line intersects each cubic function, but sometimes the related formulas generally trick the intersection values to give rise to an expression containing the root of a negative number. Educators and students know that their answers generate pictures and find intersections, and they pay attention to how to change answers using general rules of multiplication, just easy suspension of disbelief, and getting known intersections.

Unfortunately, the line of reasoning is omitted because although it doesn't spoil the math (distributive, commutative and asciative properties, and all the results of multiplication are still satisfied), it results in two positive numbers which, when multiplied to get a negative result, appear to spoil the math. And the destruction of mathematics in general frowns on mathematicians as do educators and learners. However, why is it still used? The reason the researcher understands is this: one wants to think more about the roots that arise, and the reason for thinking about that emergence is in the Cartesian plane.

Educators and students can show that numbers can be thought of as vectors on a plane, and that algebraic operations on numbers are not only visually understood on the plane but also make a difficult problem easier (getting the results of the rotation and stretching of an object). vector). Thus gradually people accept that it is elaborating the idea of "positive times positive is positive" because of thinking of multiplication in a new way, and this way does not destroy other ideas about multiplication, great visual analogies, and makes life easier in the field of interest. new to vectors.

Solving the problem typically requires obtaining an equivalent statement that simplifies the problem, explains and thus defines the meaning of the = sign. Educators and students want to get the value of x so that the statement: $2x + 4\pi + 12 = 7(x + 2\pi) - 5x - 10\pi + x$ is true. They guess the value of x and substitute it to get the statement true. It was found difficulty in that. In the discussion, the researcher asked the whole class to draw a graph of each side of the equation and look for the point of intersection of the two.

In that case, it is very easy if the statement is simplified without changing the value of x to get the statement true. That is, if $2x + 4\pi + 12 = 7(x + 2\pi) - 5x - 10\pi + x$, which is equivalent to $2(x + 2\pi) + 12 = 7(x + 2\pi) - 5(x + 2\pi) + x$, using the distributive property of the equation. Also equivalent to $2(x + 2\pi) + 12 = 2(x + 2\pi) + x$, using associative and distributive properties and arithmetic. It immediately appears that x is better than 12.

Regarding solving the division problem with fractions, which requires the concept of "inverse and multiplication", a request arose in the class, namely to ask the reason for the validity of the concept and prove it. From the expression a/b c/d, ask how many c/d are in a/b. It can be thought that c/d is as much as c of 1/d. In other words, 3/5 is three fifths, 10/7 is ten of 1/7, and so on. Thus, a/b c/d is like asking "How many c/d are in a/b?" an easy way to calculate it is to first ask about how many 1/d is in a/b."

For example, how much is 3/5 in 10/7? First, find how many fifths are in 10/7. There are 5 fifths in each unit, so there are 10/7 out of 5 fifths of a unit. How much is 11/20 in 3/2? There are twenty 1/20s in 1, so there are 20 + 10 1/20s in 1 1/2. There are 20 * 3/2 or 30 tenths in 3/2.

Then use the fact about the number of fifths in 10/7 to get how many 3/5 in 10/7? How to use the fact of how many 20s are in 3/2 to find out how many 11/20s are in 3/2? How to use knowledge of how many 1/d are in a/b to know how many c/d are in a/b? If you know that there are 50/7 fifths in 10/7, and you will get how many sets of 3 fifths are in 10/7, simply break 50/7 into groups of 3. In other words, divide 50/7 by 3. The answer is is 50/21. The answer is found by 10/7 times 5 and divided by 3, which is the same as 10/7 * 5/3.

For example 3/2 11/20. 3/2 = one whole and half. There are 20 twentieth in the whole, and 10 twentieth in half, for 30 twentieth in all. How many sets of 11 twenties are in 30 twenties? Sure 30/11. Finally, there is a/b * d 1/d and in a/b. There are c groups of 1/d in c/d. Thus there is (a/b * d)/c c/d and in a/b, or a/b c/d = (a/b * d)/c = a/b * d/c.

Regarding the ordering of numbers from largest to smallest: 0.00156; 1/60; 0.0015; 0.001; and 0.002 are still found to be difficult or take too long to do. This is related to conceptual knowledge that has not yet been formed. In conceptual understanding, for example, first that 1/60 = 100/6000 = (100/6)/1000 = (16 2/3)/1000 = 16.666666…/1000 = 1.66666…/100 = 0.1666… /10 = 0.01666…./1 = 0.016666… Thus, 0.0166666… = 1/60 > 0.002 > 0.00156 > 0.00150 > 0.0010.

Regarding multiplication as repeated addition. The discussion with the researcher was to ask educators and students to explain that the statement was false, and give examples. It is wrong if multiplication is just repeated addition. Although repeated addition can be used in calculations and conceptualizations of many types of multiplication problems. The following two examples show that repeated addition doesn't make sense. Example one: -1 * -1 = 1. Does adding -1 to -1 constitute a multiplication of any meaning? If it is an understanding, then is the result 1?

It is true that the square root of (2) * the square root of (2) = 2. Does adding something to itself is a product of an irrational number? If so, it can be defined using the distributive property as a limit: square root of (2) * 1 + square root of (2) * 4/10 + square root of (2) * 1/100 + square root of (2) * 4/1000 + … which is close to 2.

We are familiar with the order of operations to evaluate complex expressions, such as brackets and then exponents, and so on. The order of operations is either an agreement or a law. X(A + B) = XA + XB is a distributive property, which is a law. Indeed, educators and students tend to recognize that the order of operations is an agreement. In fact, consider the expression 3x + 7 = 10, which is usually written "a quantity is multiplied by three and added by seven to get a certain result. The sum is ten. This is the reason both educators and students choose to get a symbol system where multiplication and division are done before addition and subtraction, and powers are done before multiplication and division, because of the distributive nature. The researcher's thinking is that there are many languages and ways of expressing the same thoughts, there are many mathematical ways of expressing the same basic relationships.

There are different ways of expressing calculations. Computer notation, for example, recognizes the name "Back Notation" where when pressing the 3,5,+ button, the computer adds three and five. If you want to operate 3 – 4 + 5, then press 3 4 – 5 +. It doesn't require brackets. It's mathematically exactly the same, although it's just how someone asks the computer what to do on that difference.

One reason arithmetic would be different without the distributive property is that it leads us to know that -1 * -1 must equal 1. Because we know that -1 * 0 equals -1 * (something that adds up to zero when you add the first one). . Whatever notation is used to perform the first addition, it becomes clear. Thus, -1 * 0 is equivalent to adding 1 and -1 and then multiplying that sum by -1, because -1 + 1 = 0.

Using the order of operations, -1 * 0 = -1 * (1 + -1). Without knowing the distributive property, you certainly don't know that -1 * 1 + -1 * -1 is equivalent to -1 (1 + -1) and cannot be simplified or continue to calculate about -1 (1 + -1). Thus, we know that -1 * -1 + -1 * 1 = 1 * 0 = 0. And we know that -1 * 1 = -1, so -1 * -1 + -1 = 0. So, -1 * -1 must be equal to 1.

Conceptual understanding in mathematics is based on the idea of successful learning, which requires three different abilities that must be developed and nurtured simultaneously: fact control, process control, and conceptual understanding. Based on this thinking, learning should be different from learning basic skills and facts. Unfortunately, of the three types of knowledge that students need, conceptual knowledge is the most difficult to obtain. The difficulty is because knowledge is never obtained just like that, educators cannot pour concepts directly into the heads of students. On the other hand, a new concept must be based on something that students already know. That is why examples are useful when introducing new concepts. In fact, when providing abstract definitions (e.g., standard deviation is a measure of the distribution of a distribution), educators usually ask for examples (e.g., "Two groups of students have the same mean height, but one group is high and many are low, and therefore have large standard deviations, while the other groups are mostly about average height, and thus have small standard deviations”).

This is also the reason why conceptual knowledge is so important as the progress of students. Learning new concepts depends on what is already known, and as students progress, new concepts will increase depending on prior conceptual knowledge. For example, understanding algebraic equations depends on a correct conceptual understanding of the equals sign. If students fail to achieve conceptual understanding, it will become more difficult and more difficult to pursue it, because new conceptual knowledge depends on the previous (old). Learners will prefer to remember simple algorithms and use them without understanding.

The display of learning by educators that is followed by students is a characteristic of the behaviorism view. In this event, constructivism then emerged as an important epistemology in education. This form of view states that knowledge is not passively received, either through understanding or as a means of communication. Both are just meanings that are not explicitly controlled. Alternatively, constructivist practices in learning are needed where knowledge is actively built by someone who knows and requires constant adjustment. Therefore, there is no conflict between constructivists on the premise that one's knowledge changes constantly and that humans are subject to change.

Although constructivists generally view understanding as the result of an active process, they still argue that it goes beyond the naturality of the process of knowing. Knowing is simply a matter of remembering, in the form of facts or producing facts. While learning a 'new' concept reflects additions or changes to cognitive structures. Thus, the process of knowing the concept can occur from the 'bottom up' or vice versa. This kind of process links conceptual knowledge with procedural knowledge. The problem is how the two knowledges are related and how educators act as intermediaries for concept development.

Science and mathematics educators understand very well that conceptual change is just as important as concept analysis. As a matter of fact, most studies establish that concepts are mental structures of intelligence relations, not as simple as subject matter. The research in question shows that conceptual completeness composes human experience and memory (Bartsch, 1998). Therefore, conceptual change presents a structured cognitive change, not as simple as change due to the addition of facts.

Based on research in cognitive psychology, research attention in education has shifted from content (ie mathematical concepts) to mental predicates, language, and initial concepts. Despite such research, many educators continue to approach new concepts as if they simply add to their existing knowledge, namely the subject of memory and recall. Such practice is one of the causes of misconceptions in mathematics.

## V. Conclusion

That based on the current knowledge that emerges in learning, namely regarding human cognitive architecture, at a minimum guided learning is probably not effective. The facts based on research show that the so-called “constructivist” as “discovery” teaching view is wrong for two reasons: (1) Discovery or problem-based or project-based learning all do not help to unite together is to obscure the cognitive meaning of constructivist and (2) Incompatible views of inductive pedagogy confuse the needs and actions of novice experts. The consequence of the effort to implement constructivist theory is a change in emphasis, from learning a discipline of a body of knowledge to learning a discipline according to its processes and procedures.

This study shows different things about the pedagogical repertoire needed by educators. It is not meant to be direct or expository learning like everyday. Questions raised in class are pursued to become non-routine problems, original research is done, and it is said that the class puts into practice the skills learned today. Researchers do not just show conceptual and strategic thinking to transfer knowledge, but are developed to gain understanding, namely carefully designed experiences that ask students to bring and bring up previous experiences in the 'new' work requested by researchers, linking their experiences to understanding. .

In general, the expert feedback effect states that "highly effective instructional techniques with less experienced students can reduce their effectiveness and even have negative consequences when used by more experienced students". It is the knowledge of instructional techniques that begins with multiple tutoring and then fades away as the student masters it. He also demonstrates knowledge of using minimal guidance techniques to reinforce or practice previously learned material.

While it may be that practice is sufficient and the use of algorithms, learners with a little help gain a conceptual understanding of the procedures they perform. Or maybe with a strong conceptual understanding, a procedure needed to solve the problem will show clarity. Conceptual knowledge sometimes appears to precede procedural knowledge or influence its development. Also, procedural knowledge can precede conceptual knowledge. For example, students often succeed in counting before understanding all computational properties, such as sequence discrepancies.

It should be noted that the view that is accepted by most of the world of education, namely, for almost all subjects, teaches concepts or procedures for the first time. Both should be taught simultaneously and in harmony. As students gradually acquire knowledge and understanding of that knowledge, that knowledge supports understanding of others.

This study concludes that when educators teach well-structured subjects (eg arithmetic calculations, skill mapping), they use patterns such as the following: (1) Start learning with a brief review of previous learning; (2) Start the lesson with a short statement of purpose; (3) Presenting new material in small steps, providing practice for students after each step is delivered; (4) Delivering clear and in-depth teaching and explanations; (5) Provide high-level practice for all students; (6) Asking a lot of questions, checking students' understanding, and getting responses from all students; (7) Guiding students during the initial practice; (8) Provide systematic feedback and corrections; and (9) Provide clear teaching and practice for exercises in their respective seats and monitor students during work.

## References


Anderson, et al. (2001). A Taxonomy for Learning, Teaching, and Assessing: A Revisiopn of Bloom's Taxonomy of Educational Objectives. New York: David Mckay Company, Inc.

Ausubel, D.P. (1968). Educational psychology: A cognitive view. New York: Holt, Reinhart, and Winston.

Campbell, J.R., Voelkl, K.E., & Donahue, P.L. (2000). NAEP 1996 trends in academic progress (NCES 97–985r). Washington, DC: National Center for Education Statistics. Available: http://nces.ed.gov/spider/webspider/97985r.shtml. [July 10, 2001].

Davis, Leslie L. (1984). Clothing and Human Behavior. Research journal of Family & Consumer Sciences, 12 (3), 325-339

Donovan, M.S., Bransford, J.D., & Pellegrino, J.W. (Eds.). (1999). How people learn: Bridging research and practice. Washington, DC: National Academy Press. Available: http://books.nap.edu/catalog/9457.html. [July 10, 2001].

Hiebert, J., & Carpenter, T.P. (1992). Learning and teaching with understanding. In D. A.Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 65–97). New York: Macmillan.

Maher, C.A., & Martino, A.M. (1996). The development of the idea of mathematical proof: A 5-year case study. Journal for Research in Mathematics Education, 27, 194–214.

Mayer, R.E., & Hegarty, M. (1996). The process of understanding mathematical problems. In R.J.Sternberg & T.Ben-Zee (Eds.), The nature of mathematical thinking (Studies in Mathematical Thinking and Learning Series, pp. 29–53). Mahwah, NJ: Erlbaum.

Mazur, E. (2012). A visit to Harvard and Exeter: problem solving done right. ~ thoughts on education by Grant Wiggins. Tersedia: https://grantwiggins.wordpress.com/. [25 Mei 2017].

Meir Ben-Hur. (2006). Methematics Instruction: Building Strong Foundation for Reasoning and Problem Solving. Virginia: Association for Supervision and Curriculum Development.

Minsky, M. (1975). A Framework for Representing Knowledge. USA: Mc Graw Hill.

Nunes, T. (1992). Cognitive invariants and cultural variation in mathematical concepts. International Journal of Behavioral Development, 15, 433–453.

Rif'at, M. (2009). Pendidikan Matematika dari Perspektif Mengajar dan Belajar. Pontianak: Romeo Mitra Grafika.

Rittle-Johnson, B., & Alibali, M.W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? Journal of Educational Psychology, 91, 175– 189.

Schoenfeld, A.H. (1989). Explorations of students' mathematical beliefs and behavior. Journal for Research in Mathematics Education, 20, 338–355

Skemp, Richard R. (1971). The psychology of learning mathematics. Harmoodsworth: Penguin Books.

Skemp, Richard R. (1987). Intelligence, learning, and action. New York: Wiley.

Steele, C.M., & Aronson, J. (1995). Stereotype threat and the intellectual test performance of African-Americans. Journal of Personality and Social Psychology, 69, 797–811.

Tarigan, E. et al. (2020). Development of Students Work Sheet Based on Realistic Mathematic Approach with Ethnomatematic nuanced to Improve Critical Thinking of 4th Grade Students in Primary School (SD Negeri 091358 Haranggaol, Haranggaol Horisan Sub-District). Budapest International Research and Critics in Linguistics and Education (BirLE) Journal. P. 133-143.