BirEx JOURNAL
Budapest International Research in Exact Sciences
Medical, Biological, Argiculture, Engineering Science and other related areas
e-ISSN : 2655-7827
p-ISSN : 2655-7835
http://www.bircu-journal.com/index.php/birex

The Solution of a Few Infinite Problems in the Mathematics

Zhang Yue
Department of Physics, Hunan Normal University, Changsha, P. R. of China 410081.
phys_zhangyue@126.com

Abstract: *Many infinite mathematics problems take a lot of work for people, including myself, to deal with. The paper discusses a few typical infinite problems in mathematics, and it calculates out the sum of the infinite layers of the square root of 2; it calculates out the result of the problem consisting of the infinite power of; it demonstrates how to deal with the problem of the infinite decimal; it calculates out the result of the infinite continued fraction; it calculates out the result of the problem consisting of the multiplication by infinite layers of. The paper presents a method of solution commonly useful for most of the infinite problems in mathematics.*
Keywords: *infinite square root; infinite power; infinite decimal; continued fraction; infinite multiplication.*

I. Introduction

There have been a lot of studies on the problems of IMO; this is because the IMO is significant for the future of pupils in high schools, and in general, the problems of IMO are very difficult to solve. But in fact, there are many other problems which are also very difficult for people and are usually recognised as puzzling problems by pupils of high schools and even students in universities. For example, for infinite problems in mathematics [1-10], for many cases of infinite problems, most people don't know how to consider those problems. This paper will present a few infinite problems and their method.

II. Research Method

Concerning any infinite problem in mathematics, to solve it, we must change the infinite problem into a finite problem. Because there are various infinite problems in mathematics, there is no only method to solve various infinite problems in mathematics. It depends on the different problems. For example, if we want to calculate out , $\sqrt{7\times\sqrt{7\times\sqrt{7\times\sqrt{7\times......}}}}$,this is also an infinite problem in mathematics $x=\sqrt{7\times\sqrt{7\times\sqrt{7\times\sqrt{7\times......}}}}$ It is easy to find that $x=\sqrt{7x}$ the problem has become a finite problem. It is easy to solve this equation and get the result. In the following section, the paper will discuss a few other infinite problems in mathematics and demonstrate the method of dealing with various infinite problems.

III. Result and Discussion

The discussions on a few interesting infinite problems are given below:

Problem 1: Calculating the infinite square root $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+......}}}}$.

Solution: Supposing $x=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+......}}}}$, (1)

thus, eq.(1) can be written as:

Budapest International Research in Exact Sciences (BirEx) Journal
Volume 5, No 4 October 2023, Page: 211-214
e-ISSN: 2655-7827 (Online), p-ISSN: 2655-7835 (Print)
www.bircu-journal.com/index.php/birex
email: birex.journal@gmail.com

$x = \sqrt{2+x}$, (2)

it arrives $x^2 = 2 + x \; x^2 - x - 2 = 0$, (3)

the solutions of eq.(3) are given by: $x_1 = -1, x_2 = 2$. In accord with the problem, x it must be positive, $x_1 = -1$ which is wrong. Therefore, the solution to problem 1 is $x = x_2 = 2$.

Problem 2: Calculating the infinite power $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{*^{*^{*^{\cdot\cdot}}}}}}$.

Solution: Supposing $x = \sqrt{2}^{\sqrt{2}^{\sqrt{2}^{*^{*^{*^{\cdot\cdot}}}}}}$ (4)

from eq.(4), it arrives $x = (\sqrt{2})^x = (2)^{\frac{1}{2}x}$ (5)

eq.(5) can be changed into $x^{-1} = (2)^{-\frac{1}{2}x} = (2^{-1})^{\frac{1}{2}x}$ (6)

from eq.(6) it obtains

$(\frac{1}{x})^{\frac{1}{x}} = (\frac{1}{2})^{\frac{1}{2}}$ (7)

therefore $\frac{1}{x} = \frac{1}{2}$, (8)

The solution to the problem is $x = 2$.

When eq.(7) is also true, it can be proven using mathematical induction $x \le 2$. Concerning an arbitrary nth higher power, when n=1,

$x = \sqrt{2}^{\sqrt{2}} \le \sqrt{2}^{2} = 2$ (9)

supposing when n=k, , thus, when n=k+1,

$x = \sqrt{2}^{\sqrt{2}^{*^{*^{*^{\sqrt{2}(n=k+1)}}}}} \le \sqrt{2}^{\sqrt{2}^{\sqrt{2}^{*^{*^{*}}}}} = 2$ (10)

therefore, the solution to the problem can only be $x = 2$.

Problem 3: To convert the infinite decimal into a proper fraction.

Solution: Supposing $x = 0.\dot{7}$, (11)

multiplying the two sides of eq.(14) with 10, it arrives

$10x = 7 + 0.\dot{7} = 7 + x$ (12)

it is written

$9x = 7, x = \frac{7}{9}$ (13)

Therefore, the solution to problem 3 is $x = \frac{7}{9}$

Problem 4: To change the infinite decimal $0.\dot{9}\dot{8}$ into a proper fraction.

Solution: Let $x = 0.\dot{9}\dot{8}$, (14)

multiplying the two sides of eq.(14) with 100, it arrives

$100x = 98 + 0.\dot{9}\dot{8} = 98 + x$ (15)

therefore, $x = 0.\dot{9}\dot{8} = \frac{98}{99}$

Problem 5: Compare 1 with $0.\dot{9}$ which one is bigger or smaller.

Solution: At first, considering $0.\dot{9}$

let $x = 0.\dot{9}$ (16)

multiplying the two sides of eq.(16), it arrives

$10x = 9 + 0.\dot{9} = 9 + x$ (17)

It results in $9x = 9, x = 1$ (18)

therefore, $0.\dot{9} = 1$ one is not bigger or smaller than $0.\dot{9}$.

Problem 6: Calculating the infinitely continued fraction

$$\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+......}}}$$

Solution: Supposing $x = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+......}}}$, (19)

it can be written

$$x = \frac{1}{2+x} \quad (20)$$

namely, $x^2 + 2x - 1 = 0$ (21)

The solutions of eq.(19) are given by:

$x_1 = -1 - \sqrt{2}$ $x_2 = -1 + \sqrt{2}$ (22)

Following the problem, the solution must be positive. It is wrong! Therefore, the result of problem 6 is $x = x_2 = -1 + \sqrt{2}$

Problem 7: Calculating $\sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{2 \times ......}}}}$

Solution: Supposing x= $\sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{2 \times ......}}}}$ (23)

thus, from eq.(21), it arrives

X= $\sqrt{2x}$ (24)

therefore, $x^2 = 2x$ (25)

$x \cdot (x - 2) = 0$ (26)

the solution of eq.(24) is , $x_1 = 0, x_2 = 2$ but eq.(18) is not to be 0, so the result of problem 7 is $x = 2$.

IV. Conclusion

The paper discussed a few infinite problems in mathematics and physics. From the discussion, it concludes with a key idea or a common way to solve those problems. Concerning an infinite problem, at first, it can be regarded as its first bit or first part, and the other infinite part can be recognised as the same infinite problem. For example, problem one is recognised as $x = \sqrt{2 + x}$ the one on the right of the equation is the same as the one on the left. Thus, the puzzling problem becomes easy.

References

A. Gillman, P. G. Martinsson. A fast solver for Poisson problems on infinite regular lattices [J]. arXiv> math> arXiv: 1105.3505v2

A. Swiech, E. V. Teixeira. Regularity for obstacle problems in infinite dimensional Hilbert spaces [J]. Advances in Mathematics, 2009, 220 (3): 964-983.

C. G. Pinner, A. J. Van der Boorten, N. Saradha. Some infinite products with interesting continued fraction expansions [J]. Journal de theorie des numbras de Bordeaux, 5(1)(1993): 187-216.

Kamaleldin Abodayeh. Dirichlet problem in infinite networks[J]. International Journal of Pure and Applied Mathematics, 2011, 72 (3):365-374.

Kenneth O. Kortanek, Maretsugu Yamashak. Discrete infinite transportation problems[J]. Discrete Applied Mathematics, 1995, 58 (1): 19-33.

Nelo D. Alian. Infinite Decimals [J]. Rev. Bras. Hist. Mat. (Portuguese), special issue, No.1(2002):463-475.

Rutus Oldeaburger. Infinite powers of matrices and characteristic roots[J]. Duke. Math. J., 6(2)(1940):357-361.

Staples John. Real Numbers and Infinite Decimals [J]. Australian Mathematics Teacher, 29 (3)(1973): 181-184.

V. I. Skalyga. On a homotopy method in infinite-dimensional multicriteria problems [J]. Izvestiya Mathematics, 1997, 61 (4): 813-830.

Y. Kagawa, T. Yamabuchi, and Y. Araki. The infinite boundary element and its application to the unbounded Helmholtz problem [J]. The International Journal of Computation and Mathematics in Electric and Electronic Engineering, 1985, 4 (1): 19-41.