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The Application of *"Blended Learning-Rigorous Mathematical Thinking"* Model to Improve Students' Spatial Thinking Ability

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Abstract

This research is a quantitative descriptive study that explains the improvement of students' spatial thinking skills after the implementation of the blended learning-rigorous mathematical thinking model, hereinafter referred to as the BL-RMT model. Spatial thinking ability (KBSp) which is measured in this study is related to the mathematics subject in the aspect of geometry. The design of this study used a "Control Group Pretest-Postest Design" with the research sample being class VIII students at Madrasah Tsanawiyah Negeri 2 Sidoarjo. Data collection uses a test technique with an instrument in the form of questions. Data analysis using normalized Gain test (N-Gain Test) andThe t-test was performed using SPSS Independent samples test. The results showed that in the experimental class the mean value of the KBSp pre test is 68.5593 dan the average test post is 83.5593. Tthere is an increase in the average value of 15 digits and if calculated based on the N-Gain formula, a score of 0.477 is obtained in the medium Gain category. While in the control class, the mean of pre-test KBSp was 67.2581 and the mean of post-test was 75.5161. There isan increase in the average value of 8.258 and if calculated based on the N-gain formula, a score of 0.252 is obtained in the low category. Furthermore, based on the results of the t-test on the KBSp pre-test value, the result was 0.375 with a Sig (2-tailed) value of 0.708 > 0.05 ortcount < t table for = 0.05, meaning that there is no difference between the pre-test scores of the experimental class and the control class or the two groups are homogeneous.

Keywords
application; blended learning-rigorous mathematical thinking; thinking ability

I. Introduction

Learning geometry is an activity that is carried out to gain knowledge about facts, concepts, and mathematical procedures, especially those related to geometric shapes, which can then be implemented in solving everyday problems. To be able to recognize a geometric object requires the ability to think spatially about the geometrical object, because students who do not have high spatial thinking skills will find it difficult to understand geometric objects, it will be difficult to detect the relationship between geometric shapes and changes in shape. in the geometric shape, and in the end of course will also experience difficulties in solving problems related to geometrical objects. Therefore, every student must try to develop his spatial thinking skills in order to improve the ability to solve mathematical problems and other problems in everyday life.

Spatial thinking is a thinking ability that combines spatial concepts, representational tools, and reasoning processes possessed by a person in order to support problem solving abilities, especially those related to geometric shapes. In the National Academy of Science (National Academy of Science, 2006). It is stated that every useful in understanding the relations and properties in geometry to solve mathematical problems and everyday problems. Study (Hannafin, RD; Marry; Truxaw, P.; Jenniver, RV; & Yingjie. L., 2008) mentions that spatial thinking skills have a significant effect on student success when learning mathematics, students with high spatial thinking skills will have a higher success rate and are better able to solve math problems and other disciplines. Other research (Németh, 2007) states that spatial ability is really needed in engineering and mathematics, especially geometry, this ability is not found genetically but as a result of a long learning process. Based on the description and some of the research results above, it can be stated that spatial thinking skills need to be developed through learning activities to be able to assist students in improving mathematical abilities and problem solving.

The development of mathematics learning activities needs to be carried out by teachers to help students improve their ability to understand and apply the knowledge obtained in order to solve problems they face in everyday life. The development of learning in question can be in the form of applying models, methods, techniques, and using more varied media and learning teaching aids so as to minimize boredom and support learning success. One of the learning model developments carried out is the application of the BL-RMT model which combines a blended learning approach with a rigorous mathematical thinking approach. Critical analysis of the two approaches is carried out to obtain appropriate characteristics and can be used as steps for learning activities in the classroom.

This paper aims to describe the improvement of students' spatial thinking skills through the application of the development of a blended learning-rigorous mathematical thinking model. The spatial thinking ability measured in this study is certainly related to the mathematics subject in the aspect of geometry. This research is based on the hypothesis that the application of the blended learning-rigorous mathematical thinking model can improve students' spatial thinking skills.

II. Review of Literatures

2.1 Blended Learning

Blended learning is a learning approach that has the characteristics of combining conventional learning in the form of face to face with online learning assisted by information and communication technology. Blended Learning is a combination of direct learning and online learning (Khairyyah, 2021). (Thorne, 2003) stated that blended learning is the most logical evolution in learning activities that provide solutions to the development of individual needs, integrating the advances in information technology offered in online learning with the best interaction and participation in traditional learning. According to Hinneburg et al., in Syakur (2020), blended learning is a way to combine the technology and innovation offered by online learning with the interaction and participation associated with traditional learning. (Rossett, Allison; & Frazee, Rebecca, Vaughan, 2006) stated that the ideal proportion composition in blended learning is 30% online learning by utilizing information technology and the remaining 70% face-to-face learning by conducting learning interactions in the classroom through face-to-face activities.

This is different from the term blended learning model in the future because, in the future, the blended system will dominate more in learning than blended now (Hamid K, 2019). Whereas (Carman, 2005) mentions that there are 5 keys in applying blended learning approach, including: 1) live events, direct face-to-face learning in synchronous form at the same place and time; 2) online content, internet-based learning that allows students to learn independently according to their interests, time and abilities; 3) collaborations, allowing students to collaborate with each other, discuss each other, chat online and work together 4) assessment, measure student knowledge through assessment activities both before (pre-test) and after learning (post-test) and 5) reference materials, ensure that course materials have been prepared in the form of downloadable PDFs or other digital materials that students can access.

2.2 Rigorous Mathematical Thinking

Learning by applying a rigorous mathematical thinking approach is a learning that allows students to do mathematical thinking activities by maximizing their cognitive functions. (Kinard, 2006) states that mathematical thinking is a mental activity that involves rigor by combining the ability to acquire and build knowledge, apply and describe existing knowledge to gain new knowledge, transform and generalize concepts and understandings into logically interconnected ideas, facilitate problem solving in order to generate new knowledge in various contexts of human activity, utilizing cognitive processes to increase higher levels of abstraction by making critical and analytical corrections to improve understanding.

Furthermore (Kinard, James T; & Kozulin, Alex, 2008)states that the rigorous mathematical thinking approach is divided into 3 levels of cognitive functions, including: 1) Level 1: qualitative thinking as a cognitive function, the general level of thinking required by students when faced with a task or problem, is spontaneous, not systematic, not thorough and tend to be on a concrete level; 2) Level 2: quantitative thinking with accuracy as a cognitive function level of thinking based on quantitative data that can be measured and calculated such as distance, length, height, volume, speed, price, age and so on. This level requires students' ability to apply the knowledge they have in carrying out the process of calculating quantitatively so that they can solve problems; 3) Level 3: abstract logical and relational thinking in mathematical culture to draw conclusions from certain situations or problems encountered.

2.3 Model BL-RMT

The blended learning-rigorous mathematical thinking model, hereinafter referred to as the BL-RMT model, is a learning model based on a theoretical study of the blended learning approach and the rigorous mathematical thinking approach. In this model, learning activities are framed by combining the learning characteristics found in the two approaches. So that in the implementation of the BL-RMT model, it appears that there is an integration between face-to-face interactions in the classroom with online learning that utilizes advances in information and communication technology. In this BL-RMT model, it is very possible to shift the learning paradigm which was previously only teacher-centered to student-centered learning. allows for increased interaction and interactivity between students and students, students and teachers, as well as students and teachers with subject matter or other learning resources. In learning that applies the BL-RMT model, the teacher invites students to carry out mathematical thinking activities by maximizing cognitive functions that involve accuracy (rigor) so that students are able to process and build new knowledge and use it in more complex situations.

The BL-RMT model is implemented in the classroom by applying the following learning steps:

a. Literacy, the teacher facilitates students to access online the material being studied and other learning resources that are relevant to the material.
b. Stimulation, the teacher stimulates students to seek and find problems that will be discussed in learning and students respond by asking questions that are in accordance with the material discussed.
c. Problem statement, the teacher together with students determine the problems that become the focus of discussion in learning.
d. Group discussion, the teacher organizes students to discuss in independent study groups to solve the problems they face.
e. Exchange of knowledge and experience, the teacher facilitates students to exchange knowledge and experience in problem solving.
f. Assessment, the teacher evaluates the results of problem solving discussed by students both in groups and individually.

2.4 Spatial Thinking Ability

Spatial thinking is a collection of cognitive skills that combine elements of spatial concepts, representational tools, and reasoning processes in understanding objects or geometric shapes. The concept of space is related to the ability to observe and visualize geometric shapes, recognize shapes, make changes in shape and recognize changes in geometric shapes. Representation tools are related to the ability to represent shapes in the mind into real shapes or objects, for example drawing geometric shapes and converting them into real forms, sensitivity to balance, line, color, relation, shape and space. The reasoning process can include inductive reasoning and deductive reasoning. Inductive reasoning is formulating general symptoms based on things that are specific, While deductive reasoning is using general symptoms to solve specific things. The National Council of Teachers of Mathematics Standards recommends teaching mathematical concepts that can develop students' spatial thinking skills(Douglas, 2000), (NCTM, 2000), (Casey, M. Beth.; Nuttall, RL; Pezaris, E., 2001).

The three elements of spatial thinking skills, both spatial concepts, representational tools, and the reasoning process are used in understanding, applying, reasoning and compiling solutions to mathematical problems, especially those related to geometric shapes. The three elements are measured based on the results of observations during learning activities or through an assessment of the results of student work on assignment sheets and student test results sheets (National Academy of Science, 2006).

III. Research Methods

3.1 Material Objects

This research is a quantitative descriptive study that explains the improvement of students' spatial thinking skills after the implementation of the blended learning-rigorous mathematical thinking model, hereinafter referred to as the BL-RMT model. This research is important to do so that it can be seen to what extent the BL-RMT model can improve student learning outcomes, especially with regard to their spatial thinking skills. The types of data generated in this study are quantitative figures in the form of student scores obtained from observations on student test results sheets as primary data and observations on student assignment sheets as secondary data or supporting data.

3.2 Research Procedure

This study uses a "Control Group Pretest-Postest Design", which is a research design that involves two groups of students who are homogeneous and normally distributed. One group as the experimental class was then given treatment in the form of applying the BL-RMT model, while the other group as the control class was taught as usual or not given any treatment. The two groups of students were given a pretest and posttest, then observed between the class that was treated with the application of the BL-RMT model and the class whose learning was carried out as usual. The aim is to compare the conditions before and after being given treatment and also compared to the class that was not given any treatment, so that it can be known with certainty the level of significance of the treatment given.(Sugiyono, 2010).

The design of this research can be shown as the following figure.

Table 1. Control Group Pretest-Postest Design Experiment

Group	Pre-Test	Treatment	Post-Test
Experiment Class	**O1**	**X**	**O2**
Control Class	**O3**	-	**O4**

Research Design "Control Group Pretest-Posttest Design"

Informations:

X	=	Petreatment in the form of the application of the BL-RMT model
O1	=	Result of pretest experimental class before being given treatment
O2	=	Result of posttest experimental class after being given treatment
O3	=	Result of pretest control class
O4	=	Result of control class posttest

The data collected from the study was analyzed using mean and standard deviation (SD) for answering the research questions and t-test for testing the null hypotheses at probability level of 0.05. Any item with a mean value of 2.50 and above was regarded as required/Agreed/Aware.Required, while any item with a mean below 2.50 was regarded as not required/Agreed/Aware. Data analyses were carried out using SPSS version 22.0 as statistical packages.

3.3 Research Sample

The research sample for the application of the BL-RMT model was class VIII students at Madrasah Tsanawiyah Negeri 2 Sidoarjo with class VIII-A and class VIII-F as the experimental class and class VIII-E as the control class. The determination of class VIII as the research sample is adjusted to the flat-sided building material that will be used as the teaching material to be studied. The material for building a flat side is an appropriate material for measuring students' spatial thinking skills.

3.4 Data Collection Techniques and Research Instruments

The data collection technique in this study used a test technique with research instruments in the form of questions that measured students' spatial thinking skills, hereinafter referred to as the KBSp instrument. To ensure the validity of the research results, the validity and reliability tests were conducted first. The validity tests carried out include logical validity tests, namely: content, construction, and language by asking for an assessment from an expert validator (expert appraisal). Furthermore, empirical validity testing was also carried out by conducting a trial test on the KBSp instrument, and this trial was also used to calculate the level of reliability.

The results of the logical validity test of the KBSp instrument are as shown in the following table.

Table 2. Expert Validation (Expert Appraisal) KBSp . Instruments

No	Rated aspect	V1	V2	Average	Criteria
1	Content / Content	4	4	4	Tall
2	Construction	3.60	4	3.80	Tall
3	Language	4	4	4	Tall
	Aspect Average	3.87	4	3.93	Tall

While the results of the empirical trial of the KBSp instrument are as shown in the following table.

Table 3. Instrument Empirical Trial Results KBSp

Instrument	Application	Validity Test Results	Category	Reliability Test Results	Category
KBSp	Anates	0.65	Tall	0.79	Tall
	Excel	0.640645	Tall	-	-
	SPSS	thit = 61,061 (sig)	Tall	Cronbach's Alpha: 0.711	Tall

Based on the results of the validity and reliability tests above, it can be stated that the KBSp instrument has a high level of validity and reliability.

3.5 Data Analysis

Data on spatial thinking skills after the implementation of the BL-RMT model was analyzed by observing the results of student assessment tests, both pretest and posttest as primary data. Observations were also made on student assignment sheets as secondary data or supporting data. The data is then averaged and the criteria determined based on the minimum completeness criteria (KKM) value, which is 7.0 as shown in the following table.

Table 4. Spatial Thinking Ability Criteria (KBSp)

Spatial Thinking Ability (KBSp)	Criteria
.... (KBSp)< 70.0	Not enough
70.0 ≤ (KBSp) < 80.0	Enough
80.0 ≤ (KBSp) < 90.0	Well
90.0 ≤ (KBSp) ≤ 100	Very good

Furthermore, to find out the extent to which students' spatial thinking skills (KBSp) have increased, pre-test and post-test data analysis was performed using the normalized Gain test (N-Gain Test). (Meltzer, 2002), with the formula:

$$< g > = \frac{\%\ \text{actual gain}}{\%\ \text{potential gain}} = \frac{\%\ \text{posteest score} - \%\ \text{pretest score}}{100 - \%\ \text{pretest score}}$$

Criteria: 1) KBSp with "High Gain" if 0.7$\boldsymbol{g}$
2) KBSp with "Medium gain" if 0.3 $\boldsymbol{g} < 0.7$
3) KBSp with "Low gain" if $\boldsymbol{g} < 0.3$

Next, the research hypothesis was tested to determine the significance level of the implementation of the BL-RMT model for improving students' spatial thinking skills (KBSp) by conducting a t-test between the experimental class and the control class, both on pre-test scores and post-test scores. The t-test of the KBSp pre-test scores between the experimental class and the control class aims to ensure that the students' initial abilities before the implementation of the BL-RMT model are the same so that it can be guaranteed that the two classes are homogeneous. While the t-test of the KBSp post test scores between the experimental class and the control class aims to determine the differences in students' spatial thinking skills between the treated and untreated classes.

The t-test was carried out using SPSS Independent samples test, based on the steps following.

1) Meformulate a hypothesis
   H_0: ex = control : there is no difference between the experimental class and the control class
   H1: ex control : there is a difference between the experimental class and the control class
2) Determine the level of significance $\alpha = 0.05$
3) Testi statistics
   Testi statistics using SPSS version 20.0 program Independent samples test

4) Ktest criteria

H_0 accepted if tcount < ttable for$\alpha = 0.05$

H_1 accepted if tcount > ttable for$\alpha = 0.05$

IV. Discussion

4.1 Results

Data from research on spatial thinking skills (KBSp) collected include: 1) pre-test score data for both the experimental class and the control class; 2) the average between the score of the assignment sheet and the final test score which is then used as the score for the post test. Each value, both pre-test and post-test, was assigned a score criterion, and then analyzed using the N-Gains test as shown in the following table.

Table 5. Value Data KBSp with N-Gain Test

Class	Score		N-Gain	Category N-Gain
	Pre Test (Criteria)	Test Post (Criteria)		

Experiment Class (Class 8-A and 8-F)	68.5593 (not enough)	83.5593 (good)	0.477	Medium gain
Control Class (Class 8-E)	67.2581 (not enough)	75.5161 (enough)	0.252	Low gain

KBSp pre-test and post-test scores, if included in the design *Control Group Pretest-Posttest*as shown in the following table.

Table 6. KBSp Pre-Test and Post-Test Scores

Group	Pre-Test	Treatment	Post-Test
Experiment (Class 8-A and 8-F)	68.5593	**X**	83.5593
Control (Class 8-E)	67.2581	**-**	75.5161

The results of the t-test analysis on spatial thinking skills (KBSp) using SPSS version 20.0 Independent Samples Test in this study are listed in the following table.

Table 7. KBSp value t-test using SPSS version 20.0 Independent Samples Test

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Nilai Pre Tes KBSp	Equal variances assumed	.046	.830	.375	88	.708	1.30126	3.46686	-8.19092	5.58840
	Equal variances not assumed			.372	59.649	.711	1.30126	3.49658	-8.29630	5.69379
Nilai Pos Tes KBSp	Equal variances assumed	2.928	.091	4.319	88	.000	8.04319	1.86230	-11.74412	-4.34227
	Equal variances not assumed			4.654	74.814	.000	8.04319	1.72825	-11.48618	-4.60021

Based on Table 4.3 above, the results of the t-test on the KBSp pre-test value of 0.375 with a Sig (2-tailed) value of 0.708 > 0.05. It means$\mathbf{t}_{count}$ < t table forα = 0.05so that H0 is accepted and H1 is rejected, meaning that there is no difference between the experimental class and the control class and are two homogeneous class groups. While the results of the t-test on the KBSp post test value of 4.319 with a Sig (2-tailed) value of 0.000 <0.05. It means$\mathbf{t}_{count}$ > table for α = 0.05so that H0 is rejected and H1 is accepted, meaning that the two groups of students, namely the control class and the experimental class, have a significant difference in the score of the KBSp test post. So it can be stated that the application of the BL-RMT model in the experimental class can significantly improve students' spatial thinking skills (KBsp).

4.2 Discussion

The results showed that in the experimental class the mean value of the pre-test of spatial thinking skills is 68.5593 dan the average of spatial thinking ability test posts is 83.5593. Tthere is an increase in the average score of 15 digits obtained from the difference between the post-test score and the pre-test score. The difference in the average, if calculated based on the N-Gain formula, obtained an increase in students' spatial thinking skills of 0.477 with a medium Gain category.While in the control class, the mean of the pre-test of spatial thinking skills was 67.2581 and the post-test average of spatial thinking skills was 75.5161. There isan increase in the average value of 8.258 and if calculated based on the N-gain formula, the score for increasing students' spatial thinking skills is 0.252 with a low category. The calculation results based on the N-Gain analysis have actually shown differences in spatial thinking skills between the experimental class and the control class, meaning that the treatment given in the form of applying the BL-RMT model in the experimental class has a positive impact. However, this claim is still not strong, therefore it is necessary to conduct a t-test to ensure that the application of the BL-RMT model does have an effect on increasing students' spatial thinking skills.

Furthermore, based on the results of the t-test using SPSS version 20.0 Independent Samples Test on the KBSp pre-test value, the results obtained were 0.375 with a Sig (2-tailed) value of 0.708 > 0.05 or$\mathbf{t}_{count}$ < t table forα = 0.05so H0 is accepted and H1 is rejected. This means that there is no difference between the pre-test scores of the experimental class and the control class or it can be stated that the experimental class and control class are two homogeneous groups. While the results of the t-test on the value of the KBSp test post obtained the results of 4.319 with a Sig (2-tailed) value of 0.000 <0.05 or$\mathbf{t}_{count}$ > table for α = 0.05so that H0 is rejected and H1 is accepted, meaning that the spatial thinking ability in the experimental class is significantly different from the control class. Then it can be stated that the model *Blended Learning-Rigorous Mathematical Thinking* (model BL-RMT) which is applied in the experimental class is proven to improve students' spatial thinking skills.

V. Conclusion

Based on the results of the research and discussion above, it can be concluded that the application of the Blended Learning-Rigorous Mathematical Thinking model (BL-RMT model) can improve students' spatial thinking skills. The BL-RMT model as an analysis product of the blended learning approach and rigorous mathematical thinking approach has produced learning activity steps that are proven to significantly help students solve math problems and other problems in everyday life related to spatial concepts, representation tools, and reasoning process.

References

Carman, J. (2005, August). Blended learning design: Five key ingredients. Retrieved August 6, 2020, from https://www.it.iitb.ac.in/~s1000brains/rswork/dokuwiki/media/5_ingredientsofblended_learning_design.pdf

Casey, M. Beth.; Nuttall, R. L.; Pezaris, E. (2001). Spatial mechanical reasoning skills versus mathematics self confidence as mediators of gender differences on mathematics substests using cross national gender-based items. Journal for Research in Mathematics Education Volume 32, 29 - 56 Terdapat di http://www.jstor.org /stable/74962.scimag/ads.php? Doi:10.2307%2F749620.

Douglas, A. G. (2000). NCTM: Handbook of researchon mathematic teaching and learning. USA: Mac Millan Library reference.

Hamid, K. A. et.al. (2019). Evaluation of Implementation of Blanded Learning Implementation in Universitas Negeri Medan. Britain International of Linguistics, Arts and Education (BIoLAE) Journal Vol. 1 (2): 224-231.

Hannafin, R.D.; Marry; Truxaw, P.; Jenniver, R.V.; & Yingjie. L. (2008). Effects of spatial ability and instructional program on geometry achievement. Connecticut: University of Connecticut.

Khairiyyah, A., Mulyono, and Fauzi, K.M.A. (2021). The Learning Effect of Blended Learning Based on Google Class Room and Initial Mathematics on Mathematic Representation and Resilience of Students in the Covid-19 Pandemic. Britain International of Linguistics, Arts and Education (BIoLAE) Journal Vol. 3 (1): 63-76.

Kinard, J. (2006). Creating rigorous mathematical thinking: a dynamic that drives mathematical and science conceptual development. Journal of Psychology-Erdély Pszichológiai Szemle., 1-25 Tersedia di http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.121.9196&rep=rep1&type=pdf.

Kinard, James T; & Kozulin, Alex. (2008). Rigorous mathematical thinking: conceptual formation in the mathematical classroom. New York: Cambridge University Press.

Meltzer, D. (2002). The relationship between mathematics preparation andconseptual lerning gains in physics: A possible "hidden variable" in diagnostic pretest scores. Ames Lowa 50011: Department of Physics and Astronomy, Lowa Sate University.

National Academy of Science. (2006). Learning to think spatially. Washington DC: The National Academics Press.

NCTM. (2000). Principle and standards for school mathematics. Reston VA: NCTM.

Németh, B. (2007). Measurement of the development of spatial. Annales Mathematicae et Informaticae 34, pp. 123-128 Tersedia di https://ami.uni-eszterhazy.hu/uploads/papers/finalpdf/AMI_34_from123to128.pdf.

Rossett, Allison; & Frazee, Rebecca, Vaughan. (2006). Blended learning opportunities. Washington DC: American Management Association Special Report.

Syakur, A., Sugirin, Widiarni. (2020). The Effectiveness of English Learning Media through Google Classroom in Higher Education. Britain International of Linguistics, Arts and Education (BIoLAE) Journal Vol. 2 (1): 475-483.

Sugiyono. (2010). Metode Penelitian Pendidikan (Pendekatan Kuantitatif, Kualitatif, dan R&D). Bandung: Alfabeta.

Thorne, K. (2003). Blended learning: how to integrate online and traditional learning. London: Kogan Page.